Fuzzy integrated two stage vendor - buyer inventory model with crisp number

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Abstract

In today's competitive markets, close cooperation between the vendor and the buyer is necessary to reduce the joint inventory cost and the response time of the vendor-buyer system. The successful experiences of National semiconductor, Walmart and Procter and Gamble have demonstrated that integrating the supply chain has significantly influenced the company's performance and market share. This paper presents the Lagrangian method approach to determine the global optimal inventory policy for the vendor - buyer integrated system.

Keywords: Inventory model, Lagrangian method, Mean integration, Optimal policy, Vendor-buyer model.

INTRODUCTION

In today's business, a close operation is necessary to decrease the joint total inventory cost. According to Simchi-Levi et al(2000) several international companies have demonstrated that integrating the supply chain has improved the company's performance and market share. This paper considers an inventory vendor -buyer integrated system in a fuzzy situation by employing the type of fuzzy numbers which are trapezoidal. This model has been developed by using different optimization methods. A full fuzzy model is developed where the input parameters and the decision variables are 'm' fuzzified. The optimal policy for the developed model is determined by using the Lagrangian conditions after the defuzzification of the cost function with the graded mean integration methods. The proposed method finds the optimal lot size for both the vendor and buyer in an integrated two stage supply chain.

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Definition: 1. 1

A fuzzy set A, defined in the universal space X, is a function defined in X which assumes value in the range $[0,1]$.

A fuzzy set A is written as a set of pairs $\{X, A(X)\}$ as $A = \{X, A(X)\}, X$ in the set

The value is the membership grade of the element in a fuzzy set .

Definition: 1. 2

Inventory Model:
Inventory model is a mathematical model that helps business in determining the optimum level of inventories that should be maintained in a production process, managing frequency of ordering, deciding on quantity of goods or raw materials to be stored, tracking flow of supply ofraw materials and goods to provide uninterrupted service to customer without any delay in delivery.

There are two types of Inventory model widely used in business.

1 Fixed Reorder Quantity System

2 Fixed Reorder Period System.

Vendor's optimal policy

When the transfer quantity and the number of transfers has been decided by the buyer, orders are received by the vendor at known interval T_W. The vendor's overage inventory level is then obtained as

$$I_v = \frac{1}{T_v}\left\{\left[n_v Q\left(\frac{Q}{P} + \frac{(n_v-1)}{n_v}T_v\right) - \frac{n_v^2 Q^2}{2P}\right] - \left[\frac{T_v Q}{nv}\left(1+2+\cdots+(n_v-1)\right)\right]\right\}$$

Where the cycle time T_v is specified as the vendor's total profit per unit time can now be expressed as

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$$TP_v(n_v) = \frac{Cn_vQ}{T_v} - \frac{A_v}{T_v} - h_v\frac{Q}{2}\left((n_v-1)\left(1-\frac{n_vQ}{T_vP}\right)+\frac{n_vQ}{T_vP}\right) \ldots\ldots(1)$$

Substituting, $T_v = n_v n_b T_d$,

$$T_d = \frac{q^{1-\beta}}{\alpha(1-\beta)}, \mathrm{Q} = n_b\mathrm{q}$$

into expression (1) and simplifying, obtain the total profit per unit time for the vendor as

$$TP_v(n_v) = \frac{Cn_vQ}{n_vn_bT_d} - \frac{A_v}{n_vn_bT_d} - h_v\frac{n_bq}{2}\left((n_v-1)\left(1-\frac{n_vn_bq}{n_vn_bT_d}\right)+\frac{n_vn_bq}{n_vn_bT_dP}\right)$$

$$TP_v(n_v) = c\propto(1-\beta)qq^{\beta} - \frac{\alpha(1-\beta)A_v}{n_bn_vq^{1-\beta}} - h_v\frac{n_bq}{2}\left((n_v-1)+\frac{(2-n_v)\propto(1-\beta)q^{\beta}}{p}\right)$$

Taking the first and second derivatives of $TP_v(n_v)$ with respect to 1 .

$$\frac{dTP_v(n_v)}{dn_v} = \frac{\alpha(1-\beta)A_v}{n_bq^{1-\beta}n_v^2} - h_v\frac{n_bq}{2}\left(1-\frac{\alpha(1-\beta)q^{\beta}}{p}\right)$$

$$\frac{d^2TP_v(n_v)}{dn_v^2} = \frac{-2\alpha(1-\beta)A_v}{n_bq^{1-\beta}n_v^3} < 0$$

Hence, $TP_v(n_v)$ is concave in n_v. therefore, the following optimal conditions can be obtained for

$$n_v(n_v-1) \le \frac{2\alpha(1-\beta)A_vPq^{\beta}}{h_vn_b^2q^2(P-\alpha q^{\beta}(1-\beta))} \le n_v(n_v+1)$$

In the non-coordinated supply chain, the buyer chooses its own optimal policy (q^*, n_b^*) , and the vendor then chooses its optimal number of shipments n_v^*, thus, total system profit per unit time is obtained as

$$TP_N(q^*, n_b^*, n_v^*) = TP_b(q^*, n_b^*) + TP_v(n_v^*)$$

Inventory Model:

The solution space

Suppose that the problem is given by minimize y = f(x) subject to $g_i(x) \ge 0$, i=1,2,.,m.The non-negativity constraints $x \ge 0$, if any are included in the m constrains then the procedure of the extension of the Lagrangian method involves the following steps.

Step 1:

Solve the unconstrained problem:

Min $y = f(x)$

If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise set $k = 1$ and go to step2.

Step 2:

Activate any K constraint (i.e., convert them into equality) and optimize $f(x)$ subject to the active constrains by the lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all the set of active constraints taken at a time are considered without encountering the feasible solution. Go to step

Step 3:

If $k = m$, step; no feasible solution exists.

Otherwise set $K = K + 1$ and go to step 2.

GRADED MEAN INTEGRATION REPRESENTATION METHOD.

Chen and Hsieh (1999) introduced Graded mean integration representation method based on the integral values of graded mean h-level of generalized fuzzy number of for de-fuzzifying generalized fuzzy number, Hereat first generalized fuzzy number is defined as follows.

Suppose $\bar{A}$ is a generalized fuzzy number as shown in fig 1, it is described as fuzzy subset of the real line R, whose membership function $\mu_{\bar{A}}$ satisfied the following conditions,

- $\mu_{\bar{A}}(x)$ is a continuous mapping from R to $[0,1]$
- $\mu_{\bar{A}}(x) = 0, -\infty < x \le a_1$
- $\mu_{\bar{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$
- $\mu_{\bar{A}}(x) = w_a, a_2 \le x \le a_3,$
- $\mu_{\bar{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$
- $\mu_{\bar{A}}(x) = 0, a_4 \le x < \infty$, Where $0 < w_a \le 1$ and a_1, a_2, a_3, a_4 are real numbers.

This type of generalized fuzzy number is denoted as

$\bar{A} = (a_1, a_2, a_3, a_4, w_a)_{LR}$ and

$\bar{A} = (a_1, a_2, a_3, a_4, W_a)_{LR}$

when $w_a = 1$ it can be formed as

$\bar{A} = (a_1, a_2, a_3, a_4, w_a)_{LR}$ second by graded mean integration representation method. L^{-1} and R^{-1} are the inverse function of L and R respectively andgraded mean h-level values of the generalized mean h-level generalized fuzzy number is given by

$$= \frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right)$$

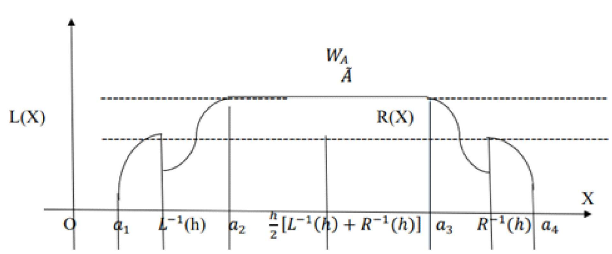

Fig.1

Then the graded mean integration representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{w_A} \frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right)dh}{\int_0^{w_A} h dh} \ldots\ldots\ldots (A)$$

Where $0 < h < w_A$ and $o < w_A \le 1$

Only use popular trapezoidal fuzzy numbers as the type of all fuzzy parameters in our purposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidalfuzzy number's and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then the Graded mean integration representation of by the formula (A) as,

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}(b_1 + b_4) + h(b_2 - b_1 - b_4 + b_3)\big] dh}{\int_0^1 h dh}$$

$$= \frac{\int_0^1 \frac{h}{2}\left[(b_1 + b_4)dh + \int_0^1 \frac{h^2}{2}(b_2 - b_1 - b_4 + b_3)\right] dh}{\int_0^1 h\, dh}$$

$$= \frac{\left[\frac{h^2}{4}\right]_0^1 (b_1 + b_4) + \left[\frac{h^3}{6}\right]_0^1 (b_2 - b_1 - b_4 + b_3)}{\left[\frac{h^2}{2}\right]_0^1}$$

$$= 2\left[\frac{1}{4}(b_1 + b_4) + \frac{1}{6}(b_2 - b_1 - b_4 + b_3)\right]$$

$$= \frac{3(b_1 + b_4) + 2(b_2 - b_1 - b_4 + b_3)}{6}$$

$$= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

FUZZY INTEGRETED TWO STAGE VENDOR - BUYER INVENTORY MODEL WITH CRISP NUMBER OF DELIVERIES AND FUZZY LOTSIZE

The fuzzy inventory integrated models by changing the crisp lot size into fuzzy lot size. Suppose fuzzy lot size $\tilde{Q}$ be a trapezoidal inventory cost of the integrated fuzzy number $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ with $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. then the fuzzy total inventory cost of the integrated two stage inventory system is,

P $(\widetilde{TC}(\tilde{Q}, \mathrm{n}))$

$$= \left\{\left(\frac{D_1A_1}{Q_4} + \frac{r_1Q_1c_{b1}}{2} + \frac{D_1s_1}{nQ_4} + \frac{r_1Q_1c_{v1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)\frac{D_1}{p_1}\right]\right),\right.$$

$$\left(\frac{D_2A_2}{Q_3} + \frac{r_2Q_2c_{b2}}{2} + \frac{D_2S_2}{nQ_3} + \frac{r_2Q_2C_{v2}}{2}\left[(n-1)\left(1-\frac{D_3}{p_2}\right)+\frac{D_2}{p_2}\right]\right),$$

$$\left(\frac{D_3A_3}{Q_2} + \frac{r_3Q_3C_{b3}}{2} + \frac{D_3S_3}{nQ_2} + \frac{r_3Q_3C_{v3}}{2}\left[(n-1)\left(1-\frac{D_2}{p_3}\right)+\frac{D_3}{p_3}\right]\right),$$

$$\left.\left(\frac{D_4A_4}{Q_1} + \frac{r_4Q_4c_{b4}}{2} + \frac{D_4S_4}{nQ_1} + \frac{r_4Q_4c_{v4}}{2} + \left[(n-1)\left(1-\frac{D_1}{p_4}\right)+\frac{D_4}{p_4}\right]\right)\right\}.$$

Then apply the Graded Mean integration representation if $P(\widetilde{TC}(\tilde{Q}, \mathrm{n}))$ by using the Graded Mean integration representation of $(\widetilde{TC}(Q, n))$ with $0 < Q_1 \le Q_2 \le Q_3 \le Q_4$. It will not change the meaning of formula if we replace inequality conditions into the following inequality .

In the following step extension of the Lagrangian method is used to find solution of Q_1, Q_2, Q_3, and Q_4 to minimize $P(\widetilde{TC}(\tilde{Q}, \mathrm{n}))$

Step 1:

Solve the constraint problem. To find the $\min P(\widetilde{TC}(\tilde{Q}, \mathrm{n}))$ with respect to Q_1, Q_2, Q_3, Q_4.

Graded Mean integration representation formula $= \frac{b_1 2b_2 2b_3 b_4}{6}$

Then using the $P(\widetilde{TC}(Q, \mathrm{n}))$ then the fuzzy total inventory cost of the integrated two stage inventory system is,

$$P\left(\widetilde{TC}(Q,\mathrm{n})\right) = \frac{1}{6}\left\{\left(\frac{D_1A_1}{Q_4} + \frac{r_1Q_1c_{b_1}}{2} + \frac{D_1S_1}{nQ_4} + \frac{r_1Q_1c_{v_1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)\frac{D_1}{p_1}\right]\right)\right.$$
$$+2\left(\frac{D_2A_2}{Q_3} + \frac{r_2Q_2C_{b2}}{2} + \frac{D_2S_2}{nQ_3} + \frac{r_2Q_2C_{v2}}{2}\left[(n-1)\left(1-\frac{D_3}{p_2}\right)+\frac{D_2}{p_2}\right]\right)$$
$$+2\left(\frac{D_3A_3}{Q_2} + \frac{r_3Q_3C_{b3}}{2} + \frac{D_3S_3}{nQ_2} + \frac{r_3Q_3C_{v3}}{2}\left[(n-1)\left(1-\frac{D_2}{p_3}\right)+\frac{D_3}{p_3}\right]\right)$$
$$\left.+\left(\frac{D_4A_4}{Q_1} + \frac{r_4Q_4c_{b4}}{2} + \frac{D_4S_4}{nQ_1} + \frac{r_4Q_4c_{v4}}{2} + \left[(n-1)\left(1-\frac{D_1}{p_4}\right)+\frac{D_4}{p_4}\right]\right)\right\}$$

$$\frac{\partial P}{\partial Q_1} = \frac{1}{6}\left\{\frac{r_1c_{b1}}{2} + \frac{r_1c_{v_1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)+\frac{D_1}{p_1}\right] - D_4A_4Q_1^{-2} - \frac{D_4S_4Q_1^{-2}}{n}\right\}$$
$$= \frac{1}{6}\left\{\frac{r_3c_{b3}}{2} + \frac{r_1c_{v1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)+\frac{D_1}{p_1}\right] - \frac{D_4A_4}{Q_1^2} - \frac{D_4S_4}{nQ_1^2}\right\}$$
$$\frac{\partial P}{\partial Q_2} = \frac{2}{6}\left\{\frac{r_2c_{b2}}{2} + \frac{r_2c_{v2}}{2}\left[(n-1)\left(1-\frac{D_3}{p_2}\right)+\frac{D_2}{p_2}\right] - D_3A_3Q_2^{-2} - \frac{D_2S_3Q_2^{-2}}{n}\right\}$$
$$= \frac{2}{6}\left\{\frac{r_2c_{b2}}{2} + \frac{r_2c_{v2}}{2}\left[(n-1)\left(1-\frac{D_3}{p_2}\right)+\frac{D_2}{p_2}\right] - \frac{D_3A_3}{Q_2^2} - \frac{D_3S_3}{nQ_2^2}\right\}$$
$$\Rightarrow \frac{\partial P}{\partial Q_3} = \frac{2}{6}\left\{\frac{r_3c_{b3}}{2} + \frac{r_3c_{v3}}{2}\left[(n-1)\left(1-\frac{D_2}{p_3}\right)+\frac{D_3}{p_3}\right] - D_2A_2Q_3^{-2} - \frac{D_2S_2Q_3^{-2}}{n}\right\}$$
$$= \frac{2}{6}\left\{\frac{r_3c_{b3}}{2} + \frac{r_3c_{v3}}{2}\left[(n-1)\left(1-\frac{D_2}{p_3}\right)+\frac{D_3}{p_3}\right] - \frac{D_2A_2}{Q_3^2} - \frac{D_2S_2}{nQ_3^2}\right\}$$
$$\frac{\partial P}{\partial Q_4} = \frac{2}{6}\left\{\frac{r_4c_{b4}}{2} + \frac{r_4c_{v_4}}{2}\left[(n-1)\left(1-\frac{D_1}{p_4}\right)+\frac{D_4}{p_4}\right] - D_1A_1Q_4^{-2} - \frac{D_1S_1Q_4^{-2}}{n}\right\}$$
$$= \frac{2}{6}\left\{\frac{r_4c_{b4}}{2} + \frac{r_4c_{v4}}{2}\left[(n-1)\left(1-\frac{D_1}{p_4}\right)+\frac{D_4}{p_4}\right] - \frac{D_1A_1}{Q_4^2} - \frac{D_1S_1Q_4^2}{nQ_4^2}\right\}$$

Let all the above partial derivatives equal to zero and solve Q_1, Q_2, Q_3, Q_4

$$\frac{\partial P}{\partial Q_1} = 0$$

$$\frac{1}{6}\left\{\frac{r_1 c_{b1}}{2}+\frac{r_1 c_{v1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)+\frac{D_1}{p_1}\right]-\frac{D_4 A_4}{Q_1^2}-\frac{D_4 S_4}{nQ_1^2}\right\}=0$$

$$\frac{r_1 c_{b1}}{2}+\frac{r_1 c_{v1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)+\frac{D_1}{p_1}\right]-\frac{D_4 A_4}{Q_1^2}-\frac{D_4 S_4}{nQ_1^2}=0$$

$$\frac{r_1 c_{b1}}{2}+\frac{r_1 c_{V_1}}{2}\left[(n-1)\left(1-\frac{D_4}{p_1}\right)+\frac{D_1}{p_1}\right]=\frac{D_4 A_4}{Q_1^2}+\frac{D_4 S_4}{nQ_1^2}$$

$$\frac{r_1 c_{b1}}{2}+\frac{r_1 c_{V_1}}{2}\left[n\left(1-\frac{D_4}{p_1}\right)+\frac{D_1+D_4}{p_1}-1\right]=\frac{D_4 A_4}{Q_1^2}+\frac{D_4 S_4}{nQ_1^2}$$

$$Q_1^2=\frac{2\left[A_4 D_4+\frac{D_4 S_4}{n}\right]}{r_1 c_{b1}+r_1 c_{v1}\left[n\left(1-\frac{D_4}{p_1}\right)+\frac{D_1+D_4}{p_1}-1\right]}$$

$$Q_1=\sqrt{\frac{2\left[A_4 D_4+\frac{D_4 S_4}{n}\right]}{r_1 c_{b1}+r_1 c_{v1}\left[n\left(1-\frac{D_4}{p_1}\right)+\frac{D_1+D_4}{p_1}-1\right]}}$$

$$\text{Similarly, } Q_2=\sqrt{\frac{2.2\left[A_3 D_3+\frac{D_3 S_3}{n}\right]}{2r_2 c_{b2}+2r_2 c_{v2}\left[\left(n\left(\frac{D_3}{p_2}\right)+\frac{D_2+D_3}{P_2}-1\right)\right]}}$$

$$Q_3=\sqrt{\frac{2.2\left[A_2 D_2+\frac{D_2 S_2}{n}\right]}{2r_3 c_{b3}+2r_3 c_{v3}\left[n\left(1-\frac{D_2}{p_3}\right)+\frac{D_2+D_3}{p_3}-1\right]}}$$

$$Q_4=\sqrt{\frac{2.2\left[A_1 D_1+\frac{D_1 S_1}{n}\right]}{2r_4 c_{b4}+2r_4 c_{v4}\left[n\left(1-\frac{D_1}{p_4}\right)+\frac{D_1+D_4}{p_4}-1\right]}}$$

Because the above show that $Q_1 > Q_2 > Q_3 > Q_4$. Therefore set $K = 1$ and go to step 2.

Step 2:

Convert the inequality constraints $Q_2 - Q_1 \geq 0$ into equality constraints $Q_2 - Q_1 = 0$ and optimize, $P(\widetilde{TC}(Q,n))$ subject to , by the Lagrangean method:

Taking the partial derivatives of with respect to and find the minimization .

Let all the above partial derivatives equal to zero and solve to . Then,

$$Q_1 = Q_2 =$$

$$\sqrt{\frac{2\left[(A_4 D_4+A_3 D_3)+\left(\frac{D_4 S_4}{n}+\frac{D_3 S_3}{n}\right)\right]}{(r_1 c_{b1}+2r_2 c_{v2})+r_1 c_{v1}\left[n\left(1-\frac{D_4}{p_1}\right)+\frac{D_1+D_4}{p_1}-1\right]+2r_2 c_{v2}\left[\left(n\left(1-\frac{D_3}{p_2}\right)+\frac{D_2+D_3}{P_2}-1\right)\right]}}$$

$$Q_3=\sqrt{\frac{2.2\left[A_2 D_2+\frac{D_2 S_2}{n}\right]}{2r_3 c_{b3}+2r_3 c_{v3}\left[n\left(1-\frac{D_2}{p_3}\right)+\frac{D_2+D_3}{p_3}-1\right]}}$$

$$Q_4=\sqrt{\frac{2.2\left[A_1 D_1+\frac{D_1 S_1}{n}\right]}{2r_4 c_{b4}+2r_4 c_{v4}\left[n\left(1-\frac{D_1}{p_4}\right)+\frac{D_1+D_4}{p_4}-1\right]}}$$

Because the above shows that Q3 > Q4 it does not satisfy the constraint $0 < Q_1 \leq Q2 \leq Q_3 \leq Q_4$. Therefore, it is not a local optimum, set k=2 and go to step 3

Step 3:

Convert the inequality constraints $Q_2 - Q_1 \geq 0, Q_3 - Q_2 \geq 0$

into equality constraints Q2 - Q1 = 0 and Q3 - Q1 = 0. Optimize, $P(\widetilde{TC}(Q, n))$ subject to Q2 - Q2 = 0 and Q3 Q2 = 0 by the Lagrangian method:

$$L(Q_1, Q_2, Q_3, Q_4, \lambda_1 \lambda_2) = P(\widetilde{TC}(Q,n)) - \lambda_1 (Q_2 - Q_1) - \lambda_2 (Q_3 - Q_2).$$

$$Q_1 = Q_2 = Q_3$$

$$= \sqrt{\frac{2[(A_4 D_4 + 2A_3 D_3 + 2A_2 D_2) + (\frac{D_4 S_4}{n} + \frac{2D_3 S_3}{n} + \frac{2D_2 S_2}{n})]}{(r_1 c_{b1} + 2 r_2 c_{b2} + 2 r_3 c_{b3}) r_1 c_{v1} + \left[n\left(1 - \frac{D_4}{p_1}\right) + \frac{D_1 + D_4}{p_1} - 1\right] + \begin{bmatrix} +2r_2 c_{v2}\left(n\left(1 - \frac{D_3}{p_2}\right) + \frac{D_2 + D_3}{p_2} - 1\right) + \\ 2r_3 c_{v3}\left[n\left(1 - \frac{D_2}{p_3}\right) + \frac{D_2 + D_3}{p_3} - 1\right] \end{bmatrix}}}$$

$$Q_4 = \sqrt{\frac{2[A_1 D_1 + \frac{D_1 S_1}{n}]}{r_4 c_{b4} + 2\, r_4 c_{v4}\, [n\left(1 - \frac{D_1}{p_4}\right) + \frac{D_1 + D_4}{p_4} - 1]}}$$

Convert the inequality constraints $Q_2 - Q_2 - Q_1 \geq 0$ $Q_3 - Q_2 \geq 0$ $Q_3 \geq 0$ into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0, Q_4 - Q_3 = 0$ Optimize, $P(\widetilde{TC}(Q,n))$ subject to and, by the Lagrangian method:

$$L(Q_1, Q_2, Q_3, Q_4, \lambda_1 \lambda_2) = P(\widetilde{TC}(Q,n)) - \lambda_1 (Q_2 - Q_1) - \lambda_2 (Q_3 - Q_2) - \lambda_3 (Q_4 - Q_3)$$

$$Q^* = Q_1 = Q_2 = Q_3 = Q_4$$

$$= \sqrt{\frac{2[(A_4 D_4 + A_3 D_3 + A_2 D_2 + A_1 D_1) + (\frac{D_4 S_4}{n} + \frac{D_3 S_3}{n} + \frac{D_2 S_2}{n} + \frac{D_1 S_1}{n})]}{(r_1 c_{b1} + 2\, r_2 c_{b2} 2\, r_3 c_{b3} + r_4 c_{b4}) + (r_1 c_{v1}\left[n\left(1 - \frac{D_4}{p_1}\right) + \frac{D_1 + D_4}{p_1} - 1\right] + \begin{bmatrix} +2r_2 c_{v2}\left(n\left(\frac{D_3}{p_2}\right) + \frac{D_2 + D_3}{p_2} - 1\right) + \\ \left[+2r_3 c_{v3} n\left(1 - \frac{D_2}{p_3}\right) + \frac{D_2 + D_3}{p_3} - 1\right] + r_4 c_{b4} + r_4 c_{v4}\left[n\left(1 - \frac{D_1}{p_4}\right) + \frac{D_1 + D_4}{p_4} - 1\right] \end{bmatrix}}}$$

Thusthe fuzzy integrated two stage vendor by inventory model is derived.

CONCLUSION

In today's business a close operation is necessary to decrease the joint total inventory cost. According to Simchi– Leviet al (2000) several international companies have demonstrated that integrating the supply chain has improved the company's performance and market share. This paper is an inventory-buyer integrated system in a fuzzy situation by employing the type of fuzzy numbers which are trapezoidal. This model has been developed by using different optimization methods.

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